Chapter 17 Solutions

17.4. Possible examples of special causes might include: wind speed and direction, traffic, temperature, Jeannine's health, or mechanical problems with the bicycle (a flat tire or a broken brake cable).

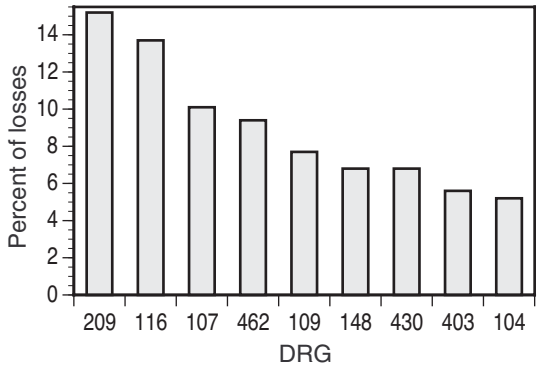
17.5. The center line is at $\mu = 168$ seconds. The control limits should be at $\mu \pm 3\sigma/\sqrt{6}$, which means about 131.26 and 204.74 seconds.

17.6. **(a)** With $n = 4$, the center line is unchanged (168 seconds), but the control limits are now $\mu \pm 3\sigma/\sqrt{4} = 123$ and 313 seconds. **(b)** With $n = 8$, the center line is unchanged (168 seconds), but the control limits are now $\mu \pm 3\sigma/\sqrt{8} \doteq 136.18$ and 199.82 seconds. **(c)** To convert to minutes, divide the original center line and control limits by 60: The center line is 2.8 minutes, and the control limits are 2.19 and 3.41 minutes.

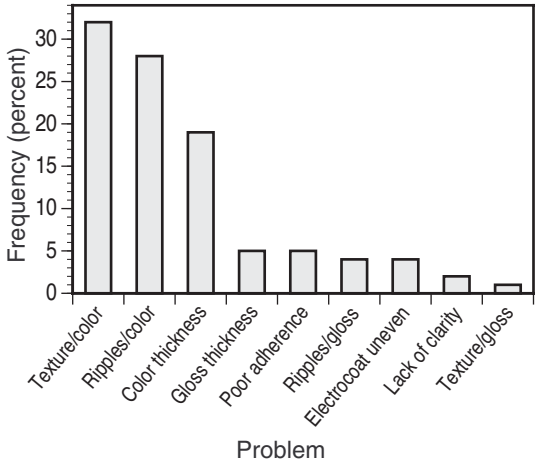
17.8. Common causes of variation might include time spent showering, getting dressed, or preparing and eating breakfast. Examples of special causes might include forgetting to set the alarm, encountering (or being in) a traffic accident, waiting for a train, or getting an unexpected phone call before leaving.

17.9. The most common problems are related to the application of the color coat; that should be the focus of our initial efforts.

For 17.9



For 17.10

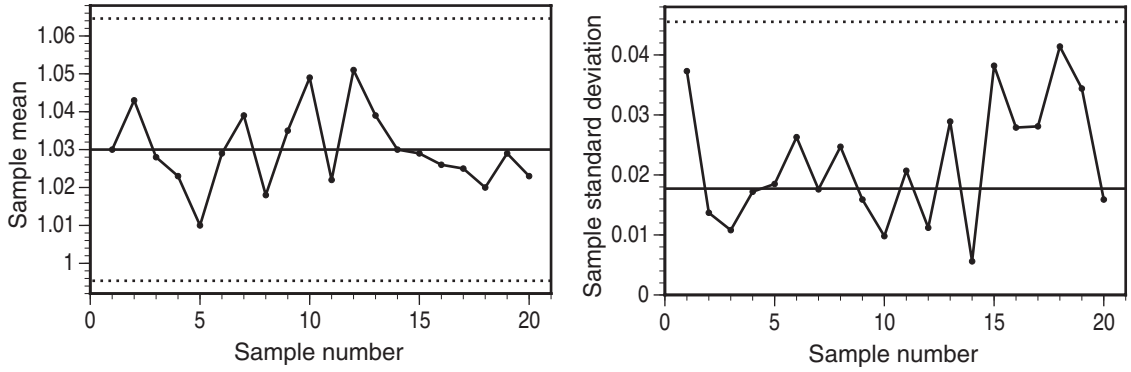


17.10. These DRGs account for a total of 80.5% of all losses. Certainly the first two (209 and 116) should be among those that are studied first; some students may also include 107, 462, and so on.

17.11. Possible causes could include alarm not set, had to wait for a train (or traffic), flat tire, spent too much time eating breakfast, and so forth.

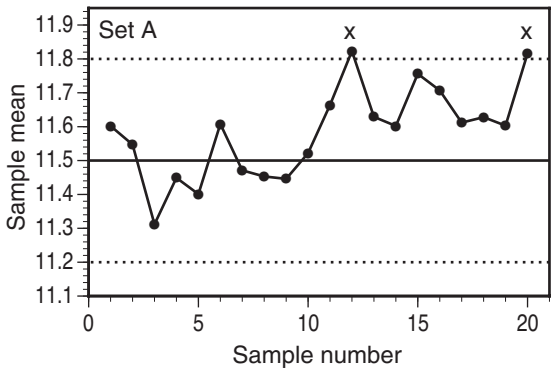
17.12. **(a)** The center line is at $\mu = 75^\circ$ F; the control limits should be at $\mu \pm 3\sigma/\sqrt{4}$, which means 74.25° F and 75.75° F. **(b)** For $n = 4$, $c_4 = 0.9213$ and $B_6 = 2.088$, so the center line for the s chart is $(0.9213)(0.5) = 0.46065^\circ$ F, and the control limits are 0 and 1.044° F.

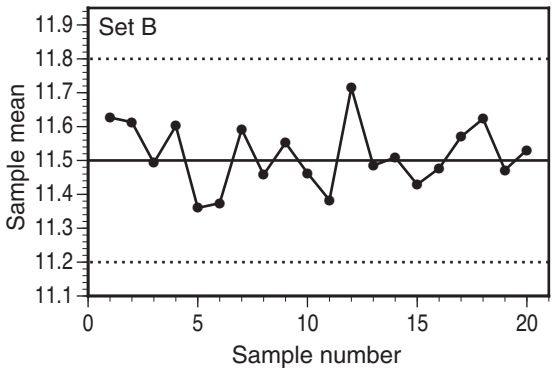
17.13. **(a)** For the $\bar{x}$ chart, the center line is at $\mu = 1.03$ lb; the control limits should be at $\mu \pm 3\sigma/\sqrt{3}$, which means 0.9954 and 1.0646 lb. **(b)** For $n = 3$, $c_4 = 0.8862$ and $B_6 = 2.276$, so the center line for the s chart is $(0.8862)(0.02) = 0.017724$ lb, and the control limits are 0 and 0.04552 lb. **(c)** The control charts are below. **(d)** Both charts suggest that the process is in control.

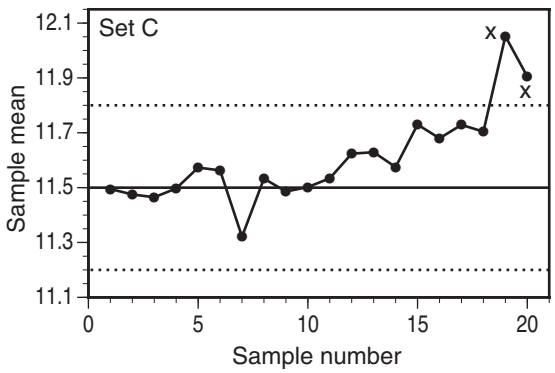


17.14. **(a)** Common causes might include processing time, normal workload fluctuation, or postal delivery time. **(b)** s-type special causes might include a new employee working in the personnel department. **(c)** Special causes affecting $\bar{x}$ might include a sudden large influx of applications or perhaps introducing a new filing system for applications.

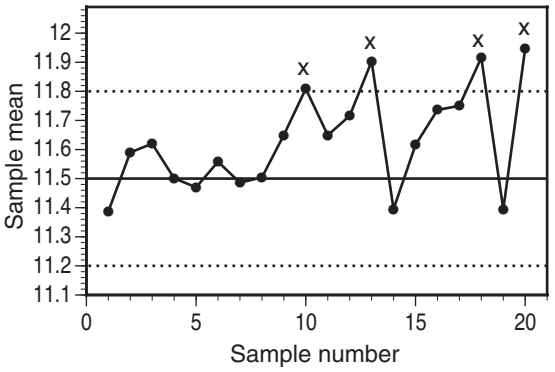
17.15. **(a)** Center: 11.5; control limits: $\mu \pm 3\sigma/\sqrt{4} = 11.5 \pm 0.3 = 11.2$ and 11.8. **(b)** Graphs on the right and below. Points outside control limits are marked with an "X." **(c)** Set B is from the in-control process. The process mean shifted suddenly for Set A; it appears to have changed on about the 11th or 12th sample. The mean drifted gradually for the process in Set C.

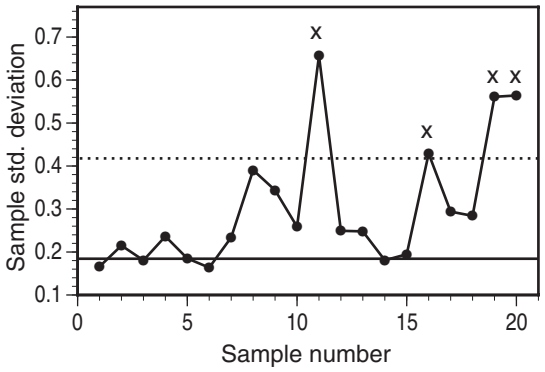






17.16. **(a)** For the $\bar{x}$ chart, the center line is 11.5, and the control limits are 11.2 and 11.8 (as in Exercise 17.15). For $n = 4$, $c_4 = 0.9213$ and $B_6 = 2.088$, so the center line for the s chart is $(0.9213)(0.2) = 0.18426$, and the control limits are 0 and 0.4176. **(b)** The s chart is certainly out of control at sample 11; the increase might have happened slightly before that point. The s chart is consistently above the center line (and often above the UCL) after that; the $\bar{x}$ chart is noticeably out of control shortly after that sample. **(c)** A change in the mean does not affect the s chart; the effect on the $\bar{x}$ chart is masked by the change in σ: Because of the increased variability, the sample means are sometimes below the center line even after the process mean shifts.

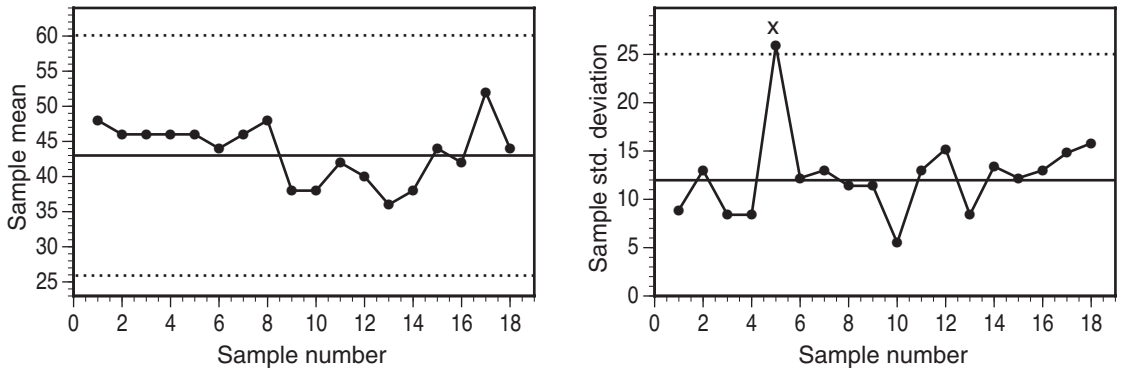




17.17. For the s chart with $n = 5$, we have $c_4 = 0.94$, $B_5 = 0$, and $B_6 = 1.964$, so the center line is 0.001128 inch, and the control limits are 0 and 0.0023568. For the $\bar{x}$ chart, the center line is $\mu = 0.8750$ inch, and the control limits are $\mu \pm 3\sigma/\sqrt{5} \doteq 0.8750 \pm 0.0016 = 0.8734$ and 0.8766 inches.

17.18. **(a)** For $n = 5$, we have $c_4 = 0.94$, $B_5 = 0$, and $B_6 = 1.964$, so the center line is 0.11938, and the control limits are 0 and 0.249428. **(b)** The center line is $\mu = 4.22$, and the control limits are $\mu \pm 3\sigma/\sqrt{5} \doteq 4.00496$ to 4.3904.

17.19. The first two means and standard deviations are $\bar{x}_1 = 48$, $s_1 = 8.9$, $\bar{x}_2 = 46$, and $s_2 = 13.0$. For the $\bar{x}$ chart, the center line is 43, and the control limits are 25.91 and 60.09. For $n = 5$, $c_4 = 0.9400$ and $B_6 = 1.964$, so the center line for the s chart is $(0.9400)(12.74) = 11.9756$, and the control limits are 0 and 25.02. The control charts (below) show that sample 5 was above the UCL on the s chart, but it appears to have been special cause variation, as there is no indication that the samples that followed it were out of control.



17.20. The new type of yarn would appear on the $\bar{x}$ chart because it would cause a shift in the mean pH. (It might also affect the process variability and therefore show up on the s chart.) Additional water in the kettle would change the pH for that kettle, which would change the mean pH and also change the process variability, so we would expect that special cause to show up on both the $\bar{x}$ and s charts.

17.21. **(a)** The process mean is the same as the center line: $\mu = 700$. The control limits are three standard errors from the mean, so $15 = 3\sigma/\sqrt{4}$, meaning that $\sigma = 10$. **(b)** If the mean changes to $\mu = 690$, then $\bar{x}$ is approximately Normal with mean 690 and standard deviation $\sigma/\sqrt{4} = 5$, so $\bar{x}$ will fall outside the control limits with probability $1 - P(685 < \bar{x} < 715) = 1 - P(-1 < Z < 5) = 0.1587$. **(c)** With $\mu = 690$ and $\sigma = 15$, $\bar{x}$ is approximately Normal with mean 690 and standard deviation $\sigma/\sqrt{4} = 7.5$, so $\bar{x}$ will fall outside the control limits with probability $1 - P(685 < \bar{x} < 715) = 1 - P(-0.67 < Z < 3.33) = 0.2518$ (software: 0.2529).

17.22. $c = 3.090$. (Looking at Table A, there appear to be three possible answers—3.08, 3.09, or 3.10. Software gives the answer 3.090232. . . .)

17.23. The usual 3σ limits are $\mu \pm 3\sigma/\sqrt{n}$ for an $\bar{x}$ chart and $(c_4 \pm 3c_5)\sigma$ for an s chart. For 2σ limits, simply replace "3" with "2." **(a)** $\mu \pm 2\sigma/\sqrt{n}$. **(b)** $(c_4 \pm 2c_5)\sigma$.

17.24. **(a)** The R chart monitors the *variability* (spread) of the process. **(b)** The R chart is commonly used because R is easier to compute (by hand) than s. **(c)** The $\bar{x}$ control limits are affected because we estimate process spread using R instead of s.

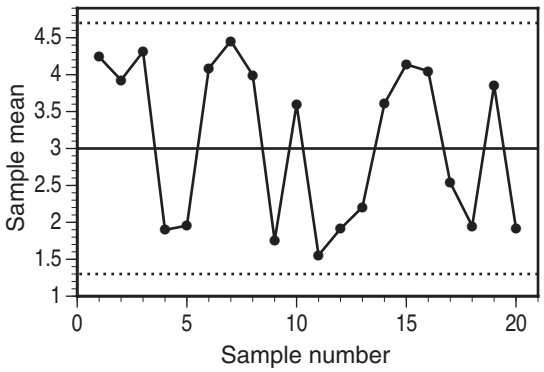
17.25. **(a)** Expanding the control limits would slow down the response; it will usually take more samples to detect an out-of-control process with 4σ control limits. **(b)** Quicker response comes at the cost of more false alarms. **(c)** The runs rule is better at detecting gradual changes. (The one-point-out rule is generally better for sudden, large changes.)

17.26. **(a)** Either (ii) or (iii), depending on whether the deterioration happens quickly or gradually. We would not necessarily expect that this deterioration would result in a change in variability (s or R). **(b)** (i) s or R chart: A change in precision suggests altered variability (s or R), but not necessarily a change in center ($\bar{x}$). **(c)** (i) s or R chart: Assuming there are other (fluent) customer service representatives answering the phones, this new person would have unusually long times, which should most quickly show up as an increase in variability. **(d)** (iii) A run on the $\bar{x}$ chart: "The runs signal responds to a gradual shift more quickly than the one-point-out signal."

17.27. We estimate $\hat{\sigma}$ to be $\bar{s}/0.9213 \doteq 1.0094$, so the $\bar{x}$ chart has center line $\bar{\bar{x}} = 48.1$ and control limits $\bar{\bar{x}} \pm 3\hat{\sigma}/\sqrt{4} \doteq 46.586$ and 49.614. The s chart has center line $\bar{s} = 0.93$ and control limits 0 and $2.088\hat{\sigma} \doteq 2.1077$.

17.28. To estimate μ and σ, we compute $\bar{\bar{x}} \doteq 1.0299$ lb and $\bar{s} \doteq 0.0222$ lb from the sample means and standard deviations given in Table 17.3. $\hat{\mu} = \bar{\bar{x}}$ is our estimate of μ; it agrees almost exactly with the historical value used in Exercise 17.13. To estimate σ, we use $\hat{\sigma} = \bar{s}/c_4 = 0.0222/0.8862 \doteq 0.0251$ lb. This is about 25% larger than the historical value $\sigma = 0.02$ lb, suggesting that the process spread *might* have increased.

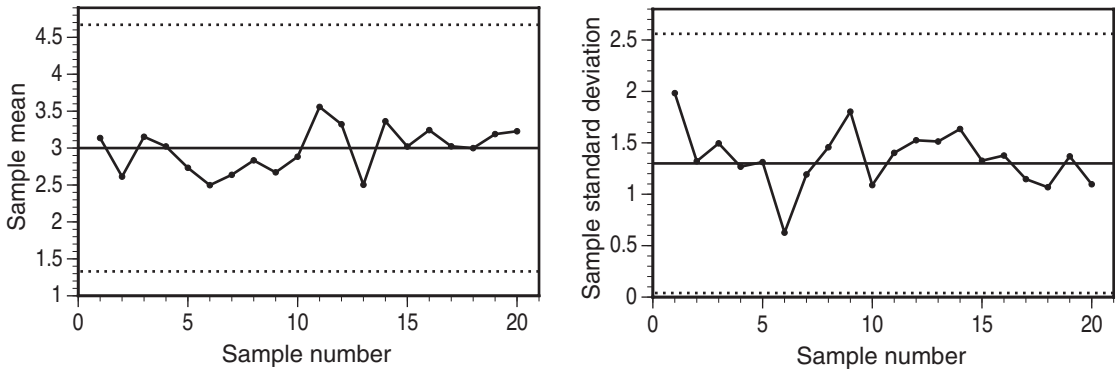
17.29. One possible $\bar{x}$ chart is shown, created with the (arbitrary) assumption that the experienced clerk processes invoices in an average of 2 minutes, while the new hire takes an average of 4 minutes. (The control limits were set arbitrarily as well.)



Note: *Such a process would not be considered to be in control for very long. The initial control limits might be developed based on a historical estimate of σ, but eventually we should assess that estimate based on our sample standard deviations. Because both clerks "are quite consistent, so that their times vary little from invoice to invoice," each sample has a small value of s, so the revised estimate of σ would likely be smaller. At that point, the control limits (based on that smaller spread) will be moved closer to the center line.*

17.30. **(a)** Sketches will vary quite a bit; many students will struggle with the implications of this situation on the appearance of the two charts. The two charts below were produced using a much more sophisticated approach than most students would take; they arose from a simulation taking samples of size 6 (3 from each clerk), where the experienced clerk's processing time (in minutes) is $N(2, 0.5)$ and the new hire's processing time is $N(4, 0.8)$. The center lines and control limits were estimated from the data. **(b)** For example, this would be acceptable if we are concerned with *overall* processing time and are not interested

in individual processing times. In particular, it would not be appropriate to compute tolerance limits for this situation because the individual measurements do not have a Normal distribution.



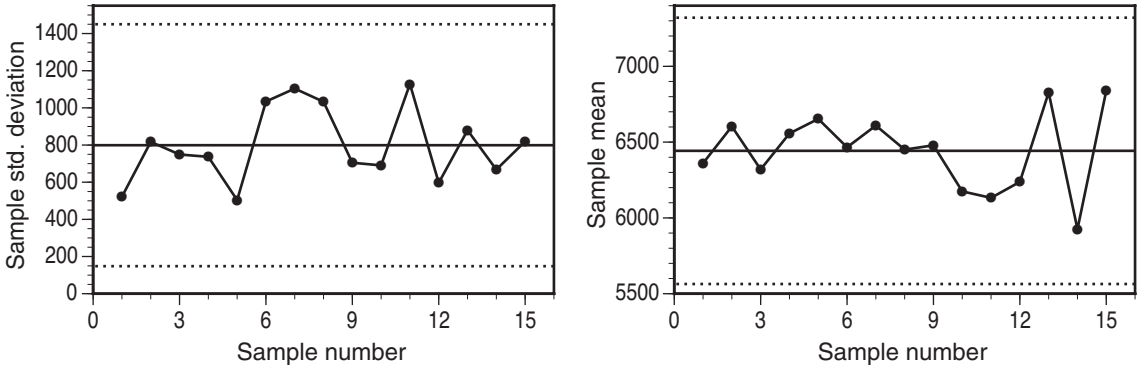
Note: *The situation described here demonstrates the problems that can arise when we do not carefully consider the question of "rational subgroups" in our sampling design; see the discussion on page 17-31.*

This situation is related to the issue of distinguishing "within-groups" variation from "between-groups" variation, as discussed in Chapter 12 (One-Way ANOVA). The within-groups variation is variation in invoice processing time for each clerk, and the between-groups variation is the difference between their processing times. In this case, though, we are not paying attention to the explanatory variable (which clerk processed the invoice), so all we see is a mixture of the two sources of variation. If—as was the case here—the two clerks were fairly consistent so that within-groups variation is small, the sample standard deviations are most affected by the between-groups variation.

Note that both charts show less variation than we typically see; nearly all the points are no more than 1 or 2 standard deviations from the center line. To begin to understand why, imagine an extreme case with no within-groups variation—where one clerk always takes exactly 2 minutes, and the other always takes exactly 4 minutes. Then each sample would contain the 6 numbers 2, 2, 2, 4, 4, and 4, so $\bar{x} = 3$ and $s \doteq 1.0954$ for all samples, and the control charts would have no variation at all.

17.31. (a) Average the 20 sample means and standard deviations and estimate μ to be $\hat{\mu} = \bar{\bar{x}} = 275.07$ and σ to be $\hat{\sigma} = \bar{s}/c_4 = 34.55/0.9213 \doteq 37.50$. **(b)** In the s chart shown in Figure 17.7, most of the points fall below the center line.

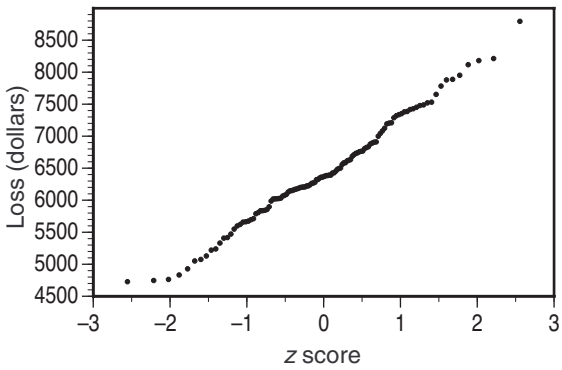
17.32. For the 15 samples, we have $\bar{s} = \$799.1$ and $\bar{\bar{x}} = \$6442.4$.
(a) $\hat{\sigma} = \bar{s}/c_4 = 799.1/0.9650 = 828.1$; the center line is $\bar{s}$, and the control limits are $B_5\hat{\sigma} = (0.179)(\$828.1) = \$148.2$ and $B_6\hat{\sigma} = (1.751)(\$828.1) = \$1450.0$. **(b)** For the $\bar{x}$ chart, the center line is $\bar{\bar{x}} = \$6442.4$, and the control limits are $\bar{\bar{x}} \pm 3\hat{\sigma}/\sqrt{8} = \$5564.1$ to $\$7320.7$. The control chart shows that the process is in control.



17.33. If the manufacturer practices SPC, that provides some assurance that the monitors are roughly uniform in quality—as the text says, "We know what to expect in the finished product." So, assuming that uniform quality is sufficiently high, the purchaser does not need to inspect the monitors as they arrive because SPC has already achieved the goal of that inspection: to avoid buying many faulty monitors. (Of course, a few unacceptable monitors may be produced and sold even when SPC is practiced—but inspection would not catch all such monitors anyway.)

17.34. The standard deviation of all 120 measurements is $s \doteq \$811.53$, and the mean is $\bar{x} \doteq \$6442.4$ (the same as $\bar{\bar{x}}$—as it must be, provided all the individual samples were the same size). The natural tolerances are $\bar{x} \pm 3s = \$4007.8$ to $\$8877.0$.

17.35. The quantile plot does not suggest any serious deviations from Normality, so the natural tolerances should be reasonably trustworthy.



Note: *We might also assess Normality with a histogram or stemplot; this looks reasonably Normal, but we see that the number of losses between \$6000 and \$6500 is noticeably higher than we might expect from a Normal distribution. In fact, the smallest and largest losses were \$4727 and \$8794. These are both within the tolerances, but note that the minimum is quite a bit more than the lower limit of the tolerances (\$4008). The large number of losses between \$6000 and \$6500 makes the mean slightly lower and therefore lowers both of the tolerance limits.*

17.36. **(a)** About 99.9% meet the old specifications: If X is the mesh tension on a randomly chosen monitor, then:

$$P(100 < X < 400) = P\left(\tfrac{100-275}{38.4} < Z < \tfrac{400-275}{38.4}\right) = P(-4.56 < Z < 3.26) \doteq 0.9994$$

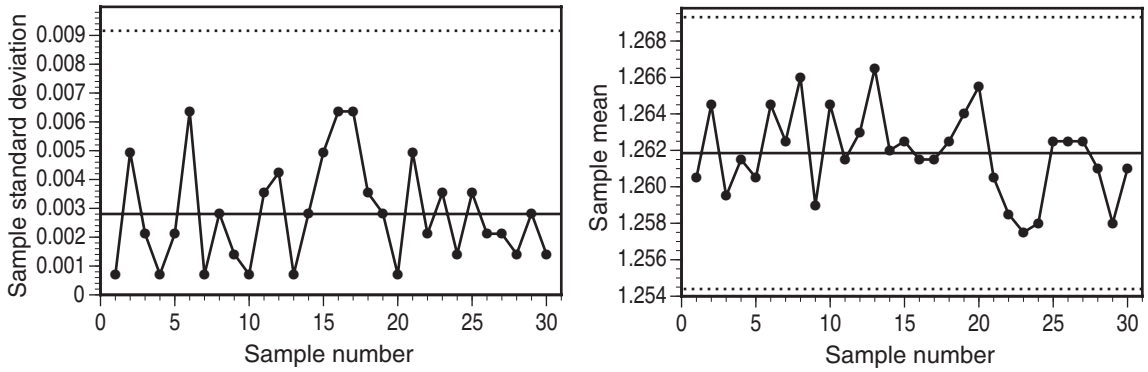
(b) About 97.4% meet the new specifications:

$$P(150 < X < 350) = P\left(\tfrac{150-275}{38.4} < Z < \tfrac{350-275}{38.4}\right) = P(-3.26 < Z < 1.95) \doteq 0.9738$$

17.37. If we shift the process mean to 250 mV, about 99% will meet the new specifications:

$$P(150 < X < 350) = P\left(\tfrac{150-250}{38.4} < Z < \tfrac{350-250}{38.4}\right) = P(-2.60 < Z < 2.60) \doteq 0.9906$$

17.38. **(a)** The means (1.2605 and 1.2645) agree exactly with those given; the standard deviations are the same up to rounding. **(b)** The s chart tracks process spread. For the 30 samples, we have $\bar{s} = 0.0028048$, so $\hat{\sigma} = \bar{s}/c_4 = \bar{s}/0.7979 \doteq 0.003515$; the center line is $\bar{s}$, and the control limits are $B_5\hat{\sigma} = 0$ and $B_6\hat{\sigma} = 2.606\hat{\sigma} \doteq 0.009161$. Short-term variation seems to be in control. **(c)** For the $\bar{x}$ chart, which monitors the process center, the center line is $\bar{\bar{x}} = 1.26185$, and the control limits are $\bar{\bar{x}} \pm 3\hat{\sigma}/\sqrt{2} \doteq 1.2544$ to 1.2693. The control chart shows that the process is in control.



17.39. The mean of the 17 in-control samples is $\bar{\bar{x}} = 43.4118$, and the standard deviation is 11.5833, so the natural tolerances are $\bar{\bar{x}} \pm 3s = 8.66$ to 78.16.

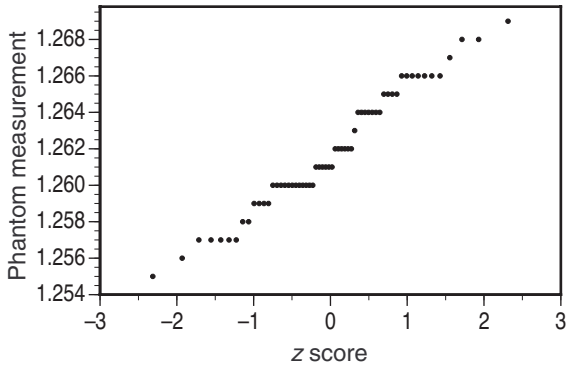
17.40. There were no out-of-control points, so we estimate the mean of the process using $\hat{\mu} = \bar{\bar{x}} = 1.26185$. The estimated standard deviation is computed from the 60 individual data points; this gives $s \doteq 0.003328$. The natural tolerances are $\bar{\bar{x}} \pm 3s = 1.2519$ to 1.2718.

17.41. Only about 44% of meters meet the specifications. Using the mean (43.4118) and standard deviation (11.5833) found in the solution to Exercise 17.39:

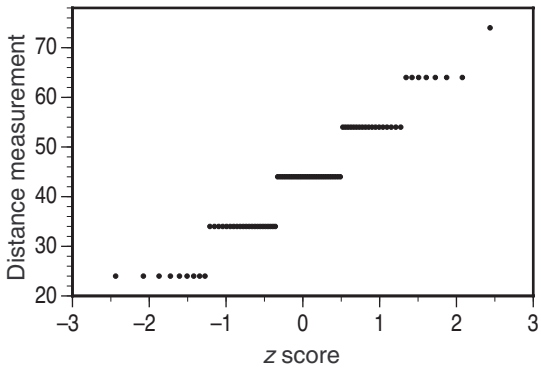
$$P(44 < X < 64) = P\left(\tfrac{44-43.4118}{11.5833} < Z < \tfrac{64-43.4118}{11.5833}\right) = P(0.05 < Z < 1.78) \doteq 0.4426$$

17.42. There is no clear deviation from Normality apart from granularity due to the limited accuracy of the recorded measurements.

For 17.42



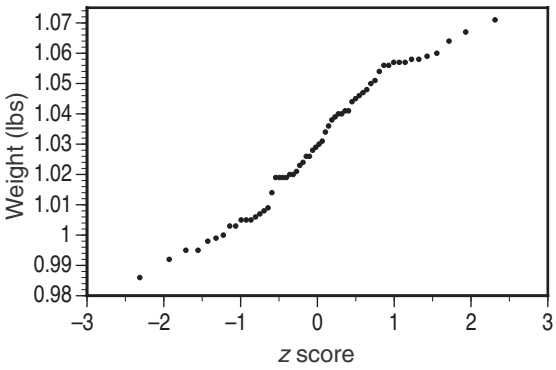
For 17.43



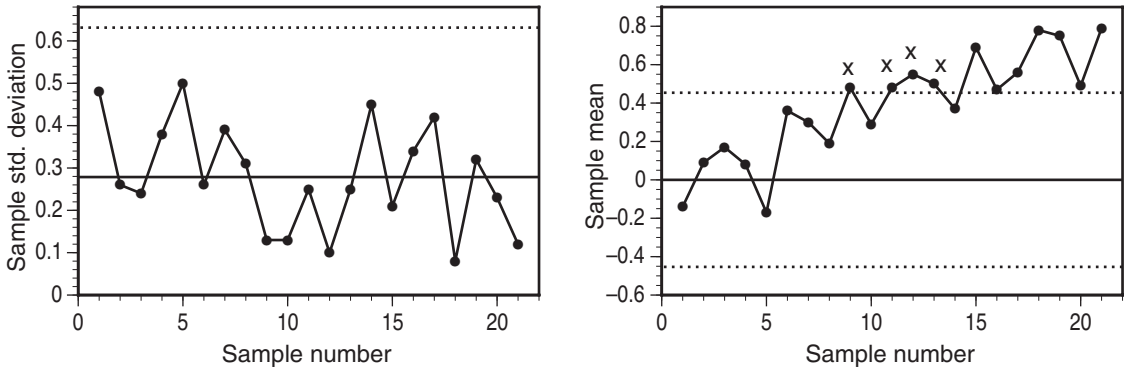
17.43. The limited precision of the measurements shows up in the granularity (stair-step appearance) of the graph. Aside from this, there is no particular departure from Normality.

17.44. The standard deviation of all 60 weights is $s \doteq 0.0224$ lb, and the mean is $\bar{x} \doteq 1.0299\overline{6}$ lb (the same as $\bar{\bar{x}}$, except for rounding error). The natural tolerances are $\bar{x} \pm 3s = 0.9627$ to 1.0972 lb.

17.45. The quantile plot, while not perfectly linear, does not suggest any serious deviations from Normality, so the natural tolerances should be reasonably trustworthy.



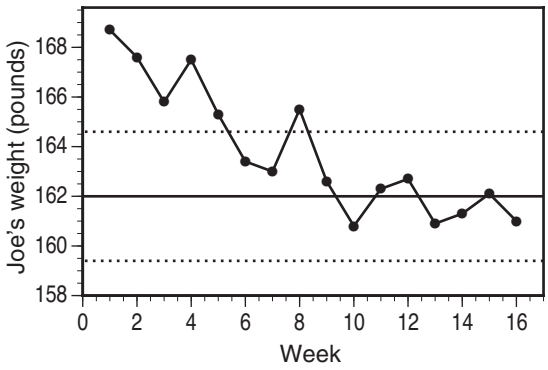
17.46. **(a)** For the 21 samples, we have $\bar{s} \doteq 0.2786$, so $\hat{\sigma} = \bar{s}/c_4 = 0.2786/0.9213 \doteq 0.3024$; the center line is $\bar{s}$, and the control limits are $B_5\hat{\sigma} = 0$ and $B_6\hat{\sigma} = (2.088)(0.3024) \doteq 0.6313$. Short-term variation seems to be in control. **(b)** For the $\bar{x}$ chart, the center line is 0 and the control limits are $\pm 3\hat{\sigma}/\sqrt{4} = \pm 0.4536$. The $\bar{x}$ chart suggests that the process mean has drifted. (Only the first four out-of-control points are marked.) One possible cause for the increase in the mean is that the cutting blade is getting dull.



17.47. **(a)** (ii) A sudden change in the $\bar{x}$ chart: This would immediately increase the amount of time required to complete the checks. **(b)** (i) A sudden change (decrease) in s or R because the new measurement system will remove (or decrease) the variability introduced by human error. **(c)** (iii) A gradual drift in the $\bar{x}$ chart (presumably a drift up, if the variable being tracked is the length of time to complete a set of invoices).

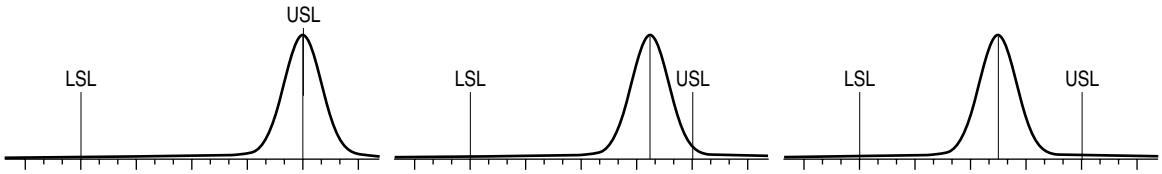
17.49. The process is no longer the same as it was during the downward trend (from the 1950s into the 1980s). In particular, including those years in the data used to establish the control limits results in a mean that is too high to use for current winning times, and a standard deviation that includes variation attributable to the "special cause" of the changing conditioning and professional status of the best runners. Such special cause variation should not be included in a control chart.

17.50. The center line is 162 pounds and the control limits are $162 \pm 2.6 = 159.4$ and 164.6 pounds. The first five points, and the eighth, are above the upper control limit; the first nine points are a "run of nine" above the center line. However, the overall impression is that Joe's weight returns to being "in control"; it decreases fairly steadily, and the last eight points are between the control limits.



17.51. LSL and USL are specification limits on the individual observations. This means that they do not apply to averages and that they are *specified* as desired output levels, rather than being *computed* based on observation of the process. LCL and UCL are control limits for the averages of samples drawn from the process. They may be determined from past data, or independently specified, but the main distinction is that the purpose of control limits is to detect whether the process is functioning "as usual," while specification limits are used to determine what percentage of the outputs meet certain specifications (are acceptable for use).

17.52. In each graph below, the large tick marks are 3σ apart, and the smaller tick marks are 1σ apart. Because $C_p = 2$, the specification limits are 12σ (4 large tick marks) apart. All three graphs could be flipped (i.e., the peak of the curve could be closer to the LSL than the USL). **(a)** $C_{pk} = 0$ means that the process mean μ falls exactly on one of the specification limits. **(b)** $C_{pk} = 0.75$ means that the nearer specification limit is $(0.75)(3\sigma) = 2.25\sigma$ above (or below) the mean. **(c)** $C_{pk} = 1.5$ means that the nearer specification limit is $(1.5)(3\sigma) = 4.5\sigma$ above (or below) the mean.



17.53. **(a)** $\hat{C}_p = \dfrac{400 - 100}{6 \times 38.38} \doteq 1.3028$ and $\hat{C}_{pk} = \dfrac{400 - 275.065}{3 \times 38.38} \doteq 1.0851$.
(b) $\hat{C}_p = \dfrac{350 - 150}{6 \times 38.38} \doteq 0.8685$ and $\hat{C}_{pk} = \dfrac{350 - 275.065}{3 \times 38.38} \doteq 0.6508$.

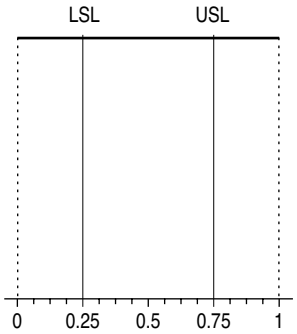
17.54. **(a)** With the original specifications, $\hat{C}_p \doteq 1.3028$ (unchanged from the previous exercise because $\hat{C}_p$ does not depend on μ) and $\hat{C}_{pk} = \dfrac{400 - 250}{3 \times 38.38} \doteq 1.3028$. **(b)** Once again, $\hat{C}_p \doteq 0.8685$ is unchanged. $\hat{C}_{pk} = \dfrac{350 - 250}{3 \times 38.38} \doteq 0.8685$.

17.55. In the solution to Exercise 17.44, we found that the mean and standard deviation of all 60 weights are $\bar{x} \doteq 1.0299\overline{6}$ lb and $s \doteq 0.0224$ lb. **(a)** $\hat{C}_p = \dfrac{1.10 - 0.94}{6 \times 0.0224} \doteq 1.1901$ and $\hat{C}_{pk} = \dfrac{1.10 - 1.03}{3 \times 0.0224} \doteq 1.0418$. (These were computed with the unrounded values of $\bar{x}$ and s; rounding will produce slightly different results.) **(b)** Customers typically will not complain about a package that was too heavy.

17.56. A change to the process mean would not change $\hat{C}_p$, but we could increase $\hat{C}_{pk}$ by centering the process mean between the specification limits, at $\mu = \frac{1.10+0.94}{2} = 1.02$ lb. With that change, $\hat{C}_{pk}$ increases to 1.1901 (the same as $\hat{C}_p$ before the change).

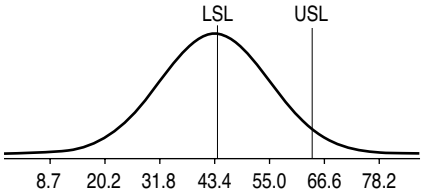
Note: *The effect of this change is hard to predict if we suspect that the weight measurements are non-Normal, but the data do not suggest any such problems (see the solution to Exercise 17.45). Additionally, decreasing the process mean might have the undesirable effect of increasing customer dissatisfaction (see part (b) of the previous exercise).*

17.57. **(a)** $C_{pk} = \frac{0.75-0.25}{3\sigma} \doteq 0.5767$. 50% of the output meets the specifications. **(b)** LSL and USL are 0.865 standard deviations above and below to mean, so the proportion meeting specifications is $P(-0.865 < Z < 0.865) \doteq 0.6130$. **(c)** The relationship between C_{pk} and the proportion of the output meeting specifications depends on the shape of the distribution.



17.58. In the solution to Exercise 17.54, we found $\hat{\sigma} = \bar{s}/c_4 \doteq 37.50$; from this, we compute $\hat{C}_{pk} = \frac{350 - 250}{3 \times 37.50} \doteq 0.8889$, which is larger than the previous value (0.8685).

17.59. See also the solution to Exercise 17.43. **(a)** Use the mean and standard deviation of the 85 remaining observations: $\hat{\mu} = \bar{x} = 43.4118$ and $\hat{\sigma} = s = 11.5833$. **(b)** $\hat{C}_p = \frac{20}{6\hat{\sigma}} \doteq 0.2878$ and $\hat{C}_{pk} = 0$ (because $\hat{\mu}$ is outside the specification limits). This process has very poor capability: The mean is too low and the spread too great. Only about 46% of the process output meets specifications.

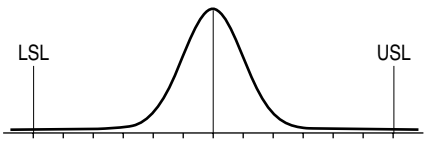


17.60. See also the solution to Exercise 17.34. **(a)** About 97.1%: For the 120 observations in Table 17.7, we find $\hat{\mu} = \bar{x} \doteq \$6442.4$ and $\hat{\sigma} = s \doteq \$811.53$. Therefore, we estimate $P(\$4500 < X < \$7500) = P(-2.39 < Z < 1.30) = 0.9032 - 0.0084 = 0.8948$. **(b)** $\hat{C}_p = \frac{7500 - 4500}{6 \times 811.53} \doteq 0.6161$. **(c)** $\hat{C}_{pk} = \frac{7500 - 6442.4}{3 \times 811.53} \doteq 0.4344$.

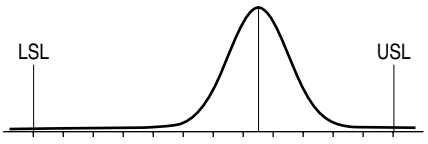
17.61. **(a)** About 97.4%: We estimate $P(14.5 < X < 15.5) = P(-2.19 < Z < 2.28) = 0.9887 - 0.0143 = 0.9744$. **(b)** $C_{pk} = \frac{14.99 - 14.5}{3s} \doteq 0.7295$.

17.62. **(a)** The number of nonconforming clips produced by an operator is (for the most part) a result of random variation within the system; no incentive can cause the operator to do better than the system allows. Part (c) offers a slightly different viewpoint. **(b)** Assuming the new machine has less variability (smaller σ), this should improve the process capability. **(c)** If some of the nonconforming clips are due to operator error, further training may have the effect of reducing σ and increasing C_{pk}. **(d)** This is unlikely to have any beneficial effect; it would result in more frequent adjustments, but these would often be unnecessary and so might degrade capability. Control limits are for correcting special-cause variation, not common-cause variation. **(e)** Better raw material should (presumably) result in better product, so this should improve the capability.

17.63. This graph shows a process with Normal output and $C_p = 2$. The tick marks are σ units apart; this is called "six-sigma quality" because the specification limits are (at least) six standard deviations above and below the mean.

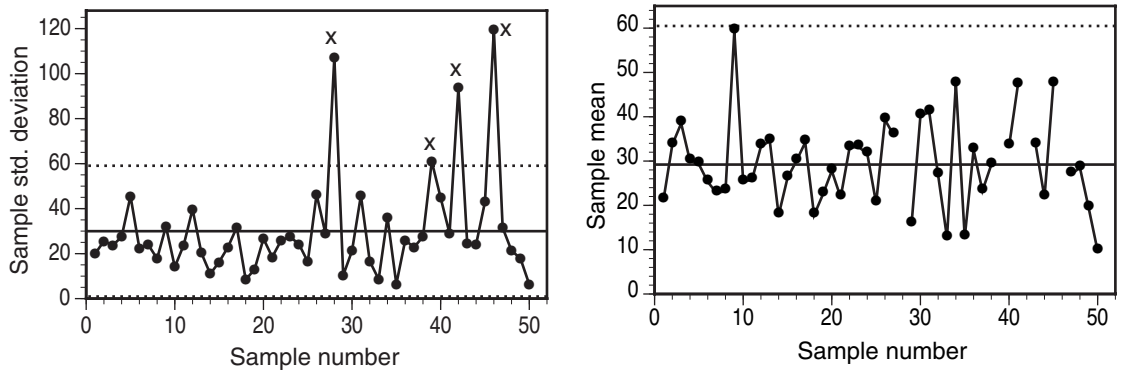


17.64. **(a)** The graph on the right shows the mean shifted toward the USL; it could also be shifted toward the LSL. As in the graph in the previous problem, tick marks are σ units apart. **(b)** $C_{pk} = \frac{4.5\sigma}{3\sigma} = 1.5$. Six-sigma quality does *not* mean that $C_{pk} \geq 2$; the latter is a stronger requirement. **(c)** The desired probability is $1 - P(-7.5 < Z < 4.5)$, for which software gives 3.4×10^{-6}, or about 3.4 out-of-spec parts per million.



17.65. Students will have varying justifications for the sampling choice. Choosing six calls per shift gives an idea of the variability and mean for the shift as a whole. If we took six consecutive calls (at a randomly chosen time), we might see additional variability in $\bar{x}$ because sometimes those six calls might be observed at particularly busy times (when a customer has to wait for a long time until a representative is available or when a representative is using the restroom).

17.66. **(a)** For $n = 6$, we have $c_4 = 0.9515$, $B_5 = 0.029$, and $B_6 = 1.874$. With $\bar{s} = 29.985$ seconds, we compute $\hat{\sigma} = \bar{s}/c_4 \doteq 31.5134$ seconds, so the initial s chart has center line $\bar{s}$ and control limits $B_5\hat{\sigma} \doteq 0.9139$ and $B_6\hat{\sigma} \doteq 59.0561$ seconds. There are four out-of-control points, from samples 28, 39, 42, and 46. **(b)** With the remaining 46 samples, $\bar{s} = 24.3015$, so $\hat{\sigma} = \bar{s}/c_4 = 25.54$ seconds, and the control limits are $B_5\hat{\sigma} = 0.741$ and $B_6\hat{\sigma} = 47.86$ seconds. There are no more out-of-control points. (The second s chart is not shown.) **(c)** We have center line $\bar{\bar{x}} = 29.2087$ seconds, and control limits $\bar{\bar{x}} \pm 3\hat{\sigma}/\sqrt{6} = -2.072$ and 60.489 seconds. (The lower control limit should be ignored or changed to 0.) The $\bar{x}$ chart has no out-of-control points.



17.67. The outliers are 276 seconds (sample 28), 244 seconds (sample 42), and 333 seconds (sample 46). After dropping those outliers, the standard deviations drop to 9.284, 6.708, and 31.011 seconds. (Sample #39, the other out-of-control point, has two moderately large times, 144 and 109 seconds; if they are removed, s drops to 3.416.)

17.68. For those 10 days, there were 961 absences and $10 \cdot 987 = 9870$ person-days available for work, so $\bar{p} = \dfrac{961}{9870} \doteq 0.09737$, and:

$$\text{CL} = \bar{p} = 0.09737, \text{ control limits: } \bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{987}} = 0.06906 \text{ and } 0.12567$$

17.69. **(a)** For those 10 months, there were 960 overdue invoices out of 28,750 total invoices (opportunities), so $\bar{p} = \dfrac{960}{28750} \doteq 0.03339$. **(b)** The center line and control limits are:

$$\text{CL} = \bar{p} = 0.03339, \text{ control limits: } \bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{2875}} = 0.02334 \text{ and } 0.04344$$

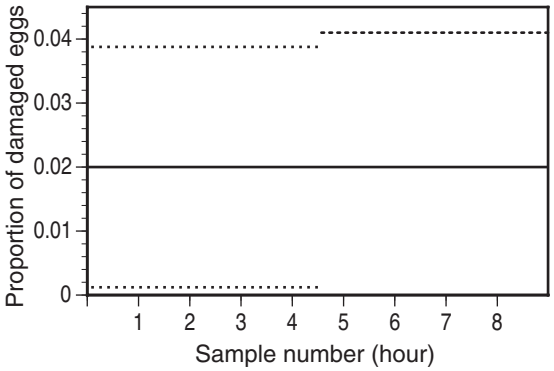
17.70. One complaint per 200 passengers means that the center line is $\bar{p} = 0.005$ (one-half of 1%), and the control limits are $\bar{p} \pm 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{1000}} = 0.005 \pm 0.0067$. As the problem says, we take LCL $= 0$, and the UCL is 0.0117.

17.71. The center line is at the historical rate (0.02); the control limits should be at $0.02 \pm 3\sqrt{\frac{0.02 \cdot 0.98}{500}}$, which means about 0.00122 and 0.03878.

17.72. For both operators, the center line is 0.02. For the first operator, the control limits are those found in the previous solution: 0.00122 and 0.03878. For the second operator, the control limits are:

$$0.02 \pm 3\sqrt{\frac{0.02 \cdot 0.98}{400}}$$

which means about -0.0010 (use 0) and 0.0410.



17.73. The center line is at $\bar{p} = \frac{214}{34{,}700} \doteq 0.006167$; the control limits should be at $\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{1530}}$, which means about 0.00016 and 0.01217.

17.74. The initial center line and control limits are:

$$\text{CL} = p = 0.01, \text{control limits: } p \pm 3\sqrt{\frac{p(1-p)}{75{,}000}} = 0.008910 \text{ and } 0.011090$$

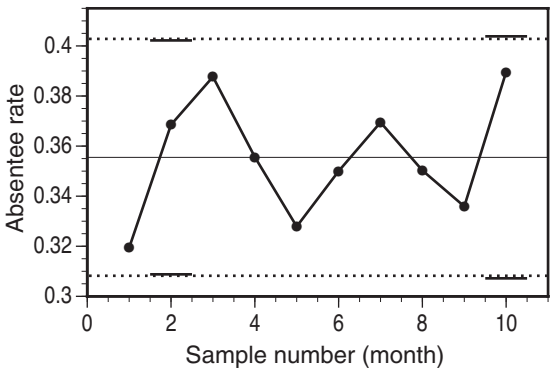
On a day when only 50,000 prescriptions are filled, the center line is unchanged, while the control limits change to:

$$p \pm 3\sqrt{\frac{p(1-p)}{50{,}000}} = 0.008665 \text{ and } 0.011335$$

17.75. **(a)** The student counts sum to 9218, while the absentee total is 3277, so $\bar{p} = \frac{3277}{9218} = 0.3555$ and $\bar{n} = 921.8$. **(b)** The center line is $\bar{p} = 0.3555$, and the control limits are:

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{921.8}} = 0.3082 \text{ and } 0.4028$$

The p chart suggests that absentee rates are in control. **(c)** For October, the limits are 0.3088 and 0.4022; for June, they are 0.3072 and 0.4038. These limits appear as solid lines on the p chart, but they are not substantially different from the control limits found in (b). Unless n varies *a lot* from sample to sample, it is sufficient to use $\bar{n}$.



17.76. **(a)** $\bar{p} = \frac{3.5}{1{,}000{,}000} = 0.0000035$. At 5000 pieces per day, we expect 0.0175 defects per day; in a 24-day month, we would expect 0.42 defects. **(b)** The center line is 0.0000035; assuming that every day we examine all 5000 pieces, the LCL is negative (so we use 0), and the UCL is 0.0000083. **(c)** Note that most of the time, we will find 0 defects, so that $\hat{p} = 0$. If we should ever find even one defect, we would have $\hat{p} = 0.0002$, and the process would be out of control. On top of this, it takes an absurd amount of testing in order to catch the rare defect.

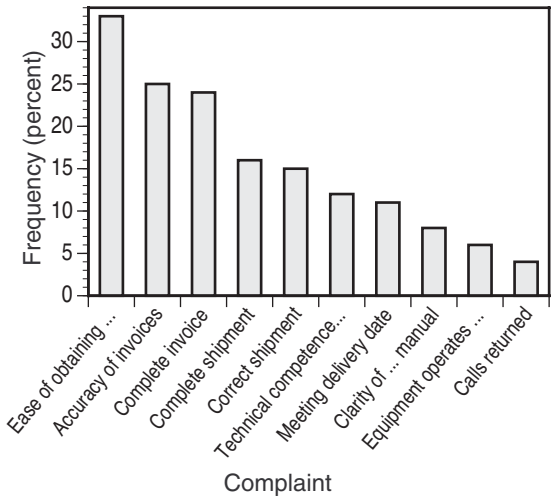
17.77. **(a)** $\bar{p} = \frac{8000}{1,000,000} = 0.008$. We expect about $4 = (500)(0.008)$ defective orders per month. **(b)** The center line and control limits are:

$$\text{CL} = \bar{p} = 0.008, \text{ control limits: } \bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{500}} = -0.00395 \text{ and } 0.01995$$

(We take the lower control limit to be 0.) It takes at least ten bad orders in a month to be out of control because $(500)(0.01995) = 9.975$.

17.78. Control charts focus on ensuring that the *process* is consistent, not that the *product* is good. An in-control process may consistently produce some percentage of low-quality products. Keeping a process in control allows one to detect shifts in the distribution of the output (which may have been caused by some correctable error); it does not help in fixing problems that are inherent to the process.

17.79. **(a)** The percents do not add to 100% because one customer might have several complaints; that is, he or she could be counted in several categories. **(b)** Clearly, top priority should be given to the process of creating, correcting, and adjusting invoices, as the three most common complaints involved invoices.

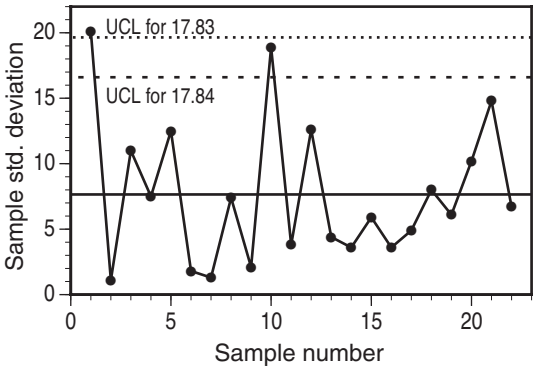


17.80. **(a)** Use $\bar{x}$ and s charts to track the time required. **(b)** Use a p chart to track the acceptance percentage. **(c)** Use a p chart to track the proportion of employees participating.

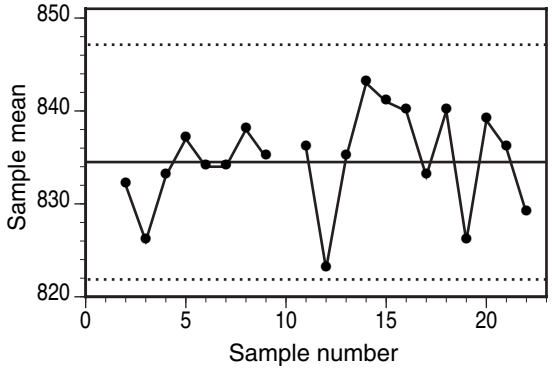
17.81. **(a)** Depending on how one interprets "availability," either $\bar{x}$ and s charts or a p chart is appropriate. **(b)** Use $\bar{x}$ and s (or R) charts to control response time. **(c)** Use a p chart for undocumented programming changes.

17.82. This situation calls for a p chart with center line $\bar{p} = \frac{3}{1000} = 0.003$ and control limits $\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{400}} = 0.003 \pm 0.0082$. We take LCL $= 0$, and the UCL is 0.0112. (In order to exceed this UCL, we would need to have 5 rejected lots out of 400.)

17.83. We find that $\bar{s} = 7.65$, so with $c_4 = 0.8862$ and $B_6 = 2.276$, we compute $\hat{\sigma} = 8.63$ and UCL $= 19.65$. One point (from sample #1) is out of control. (And, if that cause were determined and the point removed, a new chart would have s for sample #10 out of control.) The second (lower) UCL line on the control chart is the final UCL, after removing both of those samples (per the instructions in Exercise 17.84).



17.84. Without samples 1 and 10, $\bar{s} = 6.465$, $\hat{\sigma} = \bar{s}/c_4 \doteq 7.295$, and the new UCL is $2.276\hat{\sigma} = 16.60$; this line is shown on the control chart in the solution to the previous problem. Meanwhile, $\bar{\bar{x}} = 834.5$, and the control limits are $\bar{\bar{x}} \pm 3\hat{\sigma}/\sqrt{3} = 821.86$ to 847.14. The $\bar{x}$ chart gives no indication of trouble—the process seems to be in control.



17.85. **(a)** As was found in the previous exercise, $\hat{\sigma} = \bar{s}/c_4 \doteq 7.295$. Therefore, $C_p = \frac{50}{6\hat{\sigma}} \doteq 1.1423$. This is a fairly small value of C_p; the specification limits are just barely wider than the $6\hat{\sigma}$ width of the process distribution, so if the mean wanders too far from 830, the capability will drop. **(b)** If we adjust the mean to be close to 830 mm $\times 10^{-4}$ (the center of the specification limits), we will maximize C_{pk}. C_{pk} is more useful when the mean is not in the center of the specification limits. **(c)** The value of $\hat{\sigma}$ used for determining C_p was estimated from the values of s from our control samples. These are for estimating short-term variation (within those samples) rather than the overall process variation. To get a better estimate of the latter, we should instead compute the standard deviation s of the *individual* measurements used to obtain the means and standard deviations given in Table 17.11 (specifically, the 60 measurements remaining after dropping samples 1 and 10). These numbers are not available. (See "How to cheat on C_{pk}" on page 17–44 of Chapter 17.)

17.86. About 99.94%: With $\hat{\sigma} \doteq 7.295$ and mean 830, we compute $P(805 < X < 855) = P(-3.43 < Z < 3.43) = 0.9994$.

17.87. **(a)** Use a p chart, with center line $\bar{p} = \frac{15}{5000} = 0.003$ and control limits $\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{100}}$, or 0 to 0.0194. **(b)** There is little useful information to be gained from keeping a p chart: If the proportion remains at 0.003, about 74% of samples will yield a proportion of 0, and about 22% of proportions will be 0.01. To call the process out of control, we would need to see two or more unsatisfactory films in a sample of 100.

17.88. Assuming $\bar{x}$ is (approximately) Normally distributed, the probability that it would fall within the 1σ level is about 0.68, so the probability that it does this 15 times is about $0.68^{15} \doteq 0.0031$.

17.89. Several interpretations of this problem are possible, but for most reasonable interpretations, the probability is about 0.3%. From the description, it seems reasonable to assume that all three points are inside the control limits; otherwise, the one-point-out rule would take effect. Furthermore, the phrase "two out of three" could be taken to mean either "*exactly* two out of three," or "*at least* two out of three." (Given what we are trying to detect, the latter makes more sense, but students may have other ideas.)

For the kth point, we name the following events:

- A_k = "that point is no more than $2\sigma/\sqrt{n}$ from the center line,"
- B_k = "that point is 2 to 3 standard errors from the center line."

For an in-control process, $P(A_k) = 95\%$ (or 95.45%) and $P(B_k) = 4.7\%$ (or 4.28%). The first given probability is based on the 68–95–99.7 rule; the second probability (in parentheses) comes from Table A or software.

Note that, for example, the probability that the first point gives no cause for concern, but the second and third are more than $2\sigma/\sqrt{n}$ from, and on the same side of, the center line, would be:

$$\tfrac{1}{2}P(A_1 \cap B_2 \cap B_3) \doteq 0.10\% \text{ (or } 0.09\%)$$

(The factor of $1/2$ accounts for the second and third points being on the same side of the center line.) If the "other" point is the second or third point, this probability is the same, so if we interpret "two out of three" as meaning "*exactly* two out of three," then the total probability is three times the above number:

$$P(\text{false out-of-control signal from an in-control process}) \doteq 0.31\% \text{ (or } 0.26\%)$$

With the interpretation "*at least* two out of three":

$$\begin{aligned} P(\text{false out-of-control signal}) &= \tfrac{1}{2}P(A_1 \cap B_2 \cap B_3) + \tfrac{1}{2}P(B_1 \cap A_2 \cap B_3) + \tfrac{1}{2}P(B_1 \cap B_2) \\ &\doteq 0.32\% \text{ (or } 0.27\%) \end{aligned}$$